



Mansoura University
Faculty of Engineering

Final Exam.
Thursday, 11/6/2015



Faculty of Engineering

BCE, BME, CIE, MTE
Prof. Dr. Magdi S. El-Azab Mathematics 1 (MATH 101)

Time allowed: 2 hrs.

Answer the following questions (Full mark 50 pts.)

1. (a) [2 pts.] Sketch the curve $y = 2 + \ln(x - 3)$ and find its domain and range.

(b) [4 pts.] Simplify the expression: $e^{-\ln \sec(\tan^{-1} x)}$

(c) [4 pts.] Prove that $\tanh(\ln x) = \left(\frac{x^2 - 1}{x^2 + 1}\right)$.

(d) [4 pts.] Evaluate the following limits

(i) $\lim_{x \rightarrow 0^+} x^2 \ln x$

(ii) $\lim_{x \rightarrow 0^+} (1 + 4 \sin x)^{\cot x}$.

2. (a) [8 pts.] Find $y' = \frac{dy}{dx}$ if:

(i) $y = \sinh(e^{-x}) - 7^{\tan^{-1}(5x)}$,

(ii) $y = x \cos^{-1}(2x) - \frac{1}{2} \sqrt{1 - 4x^2}$

(iii) $y = \sin^3 x + \tan x^3$,

(iv) $y = x^{\tan x}$

(b) [3pts.] Solve for x the equations: $\cos^2 x - \sin^2 x = 0$.

(c) [3pts.] Show that

$$\cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right), \quad x \geq 1$$

(d) [4pts.] If $y = x \tan^{-1} x$, prove that $(1 + x^2)y'' + 2xy' = 2 + 2y$, hence, or otherwise, deduce that

$$(1 + x^2)y^{(n+2)} + 2x(n+1)y^{(n+1)} + (n+2)(n-1)y^{(n)} = 0.$$

3. (a) [3pts.] Show that the function

$$f(x) = \frac{1}{x}, \quad 1 \leq x \leq 2,$$

satisfies the hypotheses of the mean value theorem for differentiation. Then, find all points c satisfying the conclusion of the theorem.

(b) [3pts.] Find the partial fractions decomposition of the fraction

$$\frac{18x^2}{(x+2)(x-1)^2},$$

(c) [4pts.] If A is an orthogonal matrix show that AA^t is orthogonal.

(d) Given the matrices

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & 5 \\ 0 & 2 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix}$$

(i) [4pts.] Find $3A - 2C + I$, and $C^t A^t$.

(ii) [4pts.] Solve the linear system of equations $A\mathbf{x} = \mathbf{b}$.